Since the first three cases have been treated by CSI, only their results will be summarized later. The other cases are analyzed below.

(*a*) (*110*) [$\bar{1}$*12*] *Rolling*

Rolling can be considered as plane strain deformation, the strain components given by

$$\epsilon_{x'x'}=-r,\quad \epsilon_{y'y'}=0,\quad \epsilon_{z'z'}=r,$$
$$\epsilon_{y'z'}=\epsilon_{z'x'}=\epsilon_{x'y'}=0, \tag{15}$$

where x' is normal to the rolling plane, y' transverse to the rolling direction, and z' the rolling direction.

For the case of rolling on (110) along [$\bar{1}$12], let x'–[110], y'–[$\bar{1}1\bar{1}$], and z'–[$\bar{1}$12] be the coordinate axes, Fig. 3. The matrix for transformation to the cubic axes is

$$\begin{array}{c|ccc} & x' & y' & z' \\ \hline X & \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{3}} & -\dfrac{1}{\sqrt{6}} \\ Y & \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{6}} \\ Z & 0 & -\dfrac{1}{\sqrt{3}} & \dfrac{2}{\sqrt{6}}. \end{array}$$

From Eq. 15 and the transformation matrix, the strain components referred to cubic axes become

$$\epsilon_{xx}=-r/3,\quad \epsilon_{yy}=-r/3,\quad \epsilon_{zz}=2r/3,$$
$$\epsilon_{yz}=r/3,\quad \epsilon_{zx}=-r/3,\quad \epsilon_{xy}=-2r/3. \tag{16}$$

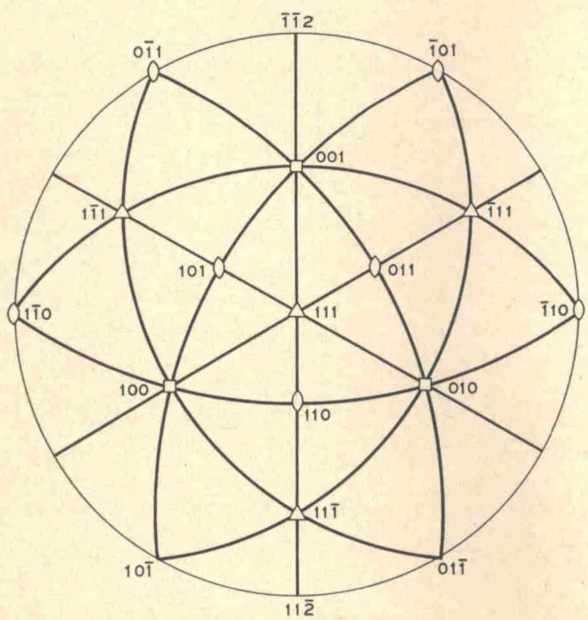

FIG. 2. Standard (111) stereographic projection of cubic crystal.

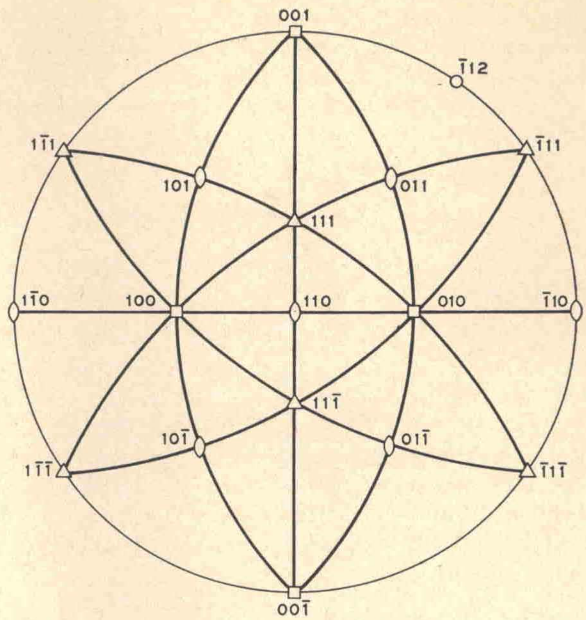

FIG. 3. Standard (110) stereographic projection of cubic crystal.

The choice of operating slip systems in rolling is complicated as the stress system is not simple. Pickus and Mathewson[14] chose those systems which have the largest value of $\cos\lambda\,\cos\theta\,\cos\varphi$, where λ and θ are the angles which the rolling plane normal makes with the slip plane normal and the slip direction, respectively; and φ is the angle between the slip direction and the rolling direction. Tucker[21] has proposed that the magnitudes of stress in rolling are σ along the rolling plane normal, $-n\sigma$ along the rolling direction, and $\frac{1}{2}(1-n)\sigma$ along the transverse direction, with $n<1$. The effective Schmid factor for a slip system is then proportional to $[(\cos\lambda\,\cos\theta)-(\cos\gamma\,\cos\varphi)]$, where γ is the angle between the slip plane normal and the rolling direction. The other symbols retain their previous meanings. Those slip systems with the largest effective Schmid factor will then operate. Although both criteria often lead to the choice of the same slip systems, it was felt that the Tucker approach is more appropriate and hence adopted here. In any event, the operating slip systems based on either stress criterion may be insufficient to accommodate the macroscopic strains. In such a case, additional slip on other systems may be required.[22]

From the stereographic projection of Fig. 3, the most likely slip systems that operate during (110) [$\bar{1}$12]

[21] G. E. G. Tucker, Acta Met. **12**, 1093 (1964).

[22] In the Bishop and Hill analysis,[12,13] one can obtain a sufficient number of slip systems to accommodate the imposed macroscopic strains while satisfying the yield criterion, i.e., the resolved shear stress for slip is reached equally in all the operating slip systems. With reference to the operating slip systems analyzed in the present paper using the Tucker approach, a recent calculation[23] based on the Bishop and Hill analysis yields the same results.

[23] G. Y. Chin, E. A. Nesbitt, and A. J. Williams, Acta Met. (to be published).

rolling are (111) [10$\bar{1}$] and (11$\bar{1}$) [011] which are in accord with both the criteria of Tucker and of Pickus and Mathewson. These are systems Nos. (2) and (5) according to Table I. The strain components in terms of slip density are:

$$2\epsilon_{xx}=-S_2,\quad 2\epsilon_{yy}=S_5,\quad 2\epsilon_{zz}=S_2-S_5,$$
$$4\epsilon_{yz}=S_2,\quad 4\epsilon_{zx}=S_5,\quad 4\epsilon_{xy}=-S_2+S_5. \quad (17)$$

Solution of Eqs. (16) and the normal strain equations of (17) gives

$$S_2=\tfrac{2}{3}r,\quad S_5=-\tfrac{2}{3}r, \quad (18)$$

which, however, do not satisfy the shear-strain equations of (17). In this case, other slip systems may be forced to act. Since [011] and [10$\bar{1}$] are the only pair of slip directions symmetrical to the specimen coordinate axes, the most likely slip systems to act are (1$\bar{1}$1) [10$\bar{1}$] and (1$\bar{1}$1) [011] [Nos. (8) and (9), respectively] in addition to Nos. (2) and (5). The strain components then become

$$2\epsilon_{xx}=-S_2-S_8,$$
$$2\epsilon_{yy}=S_5-S_9,$$
$$2\epsilon_{zz}=S_2-S_5+S_8+S_9, \quad (19)$$
$$4\epsilon_{yz}=S_2-S_8,$$
$$4\epsilon_{zx}=S_5+S_9,$$
$$4\epsilon_{xy}=-S_2+S_5+S_8+S_9.$$

Solution of Eqs. (16) and (19) gives

$$S_2=r,\quad S_5=-r,\quad S_8=S_9=-(r/3) \quad (20)$$

with all the strain components satisfied. It may be noted that systems (8) and (9) are in cross-slip relationship (sharing the same slip direction) with systems (2) and (5), respectively, and that the amount of slip required of (8) and (9) is only one-third that of (2) and (5).

Although the Schmid factor is zero for systems (8) and (9), local stress variations may generate sufficient slip to satisfy the macrostrains. Alternatively, slip may still occur predominantly in systems (2) and (5) with resultant shape changes other than those prescribed by Eq. (15). Both modes of deformation are considered below.

On the assumption that only slip systems (2) and (5) operate, the induced anisotropy energy for L.F. deformation, according to Eqs. (2) and (18) and Table I, is

$$E_{\mathrm{LF}}=\tfrac{1}{8}K_{\mathrm{LF}}[\tfrac{2}{3}r(\tfrac{1}{2}\alpha_1^2+\tfrac{1}{2}\alpha_2^2+\alpha_3^2-\alpha_3\alpha_2+\alpha_3\alpha_1)]$$
$$=(1/24)K_{\mathrm{LF}}r(\alpha_3^2-2\alpha_3\alpha_2+2\alpha_3\alpha_1)+\text{const.} \quad (21)$$

For magnetic torque and hysteresis loop measurements, it is more convenient to confine the anisotropy to the rolling plane. On the (110) rolling plane, Eq. (21) indicates that the easy direction is [$\bar{1}$11], which is 19.5° from the [$\bar{1}$12] rolling direction, Fig. 3.

For S.C. deformation, Eqs. (3) and (20) and Table I give

$$E_{\mathrm{SC}}=\tfrac{1}{16}K_{\mathrm{SC}}(\tfrac{2}{3}r)(\tfrac{2}{3}\alpha_1\alpha_2)=(1/36)K_{\mathrm{SC}}r\alpha_1\alpha_2. \quad (22)$$

E_{SC} is thus minimum along [$\bar{1}$10] which is 55° from the [$\bar{1}$12] rolling direction. If slip systems (2), (5), (8), and (9) all operate,

$$E_{\mathrm{LF}}=(5/48)K_{\mathrm{LF}}r(\alpha_3^2-2\alpha_3\alpha_2+2\alpha_3\alpha_1)+\text{const.}, \quad (23)$$

which again places the easy direction along [$\bar{1}$11]. For S.C. deformation,

$$E_{\mathrm{SC}}=(1/24)K_{\mathrm{SC}}r\alpha_1\alpha_2$$
$$+(1/72)K_{\mathrm{SC}}r(-\alpha_2\alpha_3+\alpha_3\alpha_1-\alpha_1\alpha_2)$$
$$=(1/72)K_{\mathrm{SC}}r(2\alpha_1\alpha_2-\alpha_2\alpha_3+\alpha_3\alpha_1). \quad (24)$$

A calculation based on Eq. (24) shows that on the (110) plane, the easy direction is near [$\bar{1}$11] and about 25° from the [$\bar{1}$12] rolling direction.

(b) (110) [$\bar{1}$10] Rolling

Let x'–[110], y'–[00$\bar{1}$], and z'–[$\bar{1}$10] be the coordinate axes, Fig. 3. The matrix for the transformation to cubic axes is

	x'	y'	z'
x	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$
y	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$
z	0	−1	0.

From Eq. (15) and the transformation matrix, the strain components referred to cubic axes become

$$\epsilon_{xx}=\epsilon_{yy}=\epsilon_{zz}=\epsilon_{yz}=\epsilon_{zx}=0,\quad \epsilon_{xy}=-r. \quad (25)$$

The most likely slip systems to operate are Nos. (1), (2), (4), (5), (8), (9), (10), and (11) according to the Tucker criterion. [Only (1), (2), (4), and (5) are chosen according to the criterion of Pickus and Mathewson.] In this case

$$2\epsilon_{xx}=-S_2+S_4-S_8-S_{11},$$
$$2\epsilon_{yy}=-S_1+S_5-S_9-S_{10},$$
$$2\epsilon_{zz}=S_1+S_2-S_4-S_5+S_8+S_9+S_{10}+S_{11}, \quad (26)$$
$$4\epsilon_{yz}=S_2+S_4-S_8+S_{11},$$
$$4\epsilon_{zx}=S_1+S_5+S_9-S_{10},$$
$$4\epsilon_{xy}=-S_1-S_2+S_4+S_5+S_8+S_9+S_{10}+S_{11}.$$

Solution of Eqs. (25) and (26) gives

$$S_1=S_2=r/2,\ S_4=S_5=S_8=S_9=S_{10}=S_{11}=-(r/2), \quad (27)$$

with the result that

$$E_{\mathrm{LF}}=\tfrac{1}{8}K_{\mathrm{LF}}r\alpha_3^2+\text{const.} \quad (28)$$

and

$$E_{\mathrm{SC}}=0. \quad (29)$$